RESEARCH ARTICLE OPEN ACCESS

Interactive Fuzzy Goal Programming approach for Tri-Level Linear Programming Problems

Dr. Taghreed A. Hassan.*
* Department of Basic Engineering science, Faculty of Engineering, Shebin El-Kom, Minufia University, Egypt

Abstract
The aim of this paper is to present an interactive fuzzy goal programming approach to determine the preferred compromise solution to Tri-level linear programming problems considering the imprecise nature of the decision makers' judgments for the objectives. Using the concept of goal programming, fuzzy set theory, in combination with interactive programming, and improving the membership functions by means of changing the tolerances of the objectives provide a satisfactory compromise (near to ideal) solution to the upper level decision makers.

Two numerical examples for three-level linear programming problems have been solved to demonstrate the feasibility of the proposed approach. The performance of the proposed approach was evaluated by using of metric distance functions with other approaches.

Keywords: interactive programming, goal programming, multi-objective optimization, fuzzy set theory and tri-level decision making,

I. Introduction:

Hierarchical decision making is strongly motivated by real-world applications. These applications can be formulated within a bi-level programming (BLP) problem framework, where an upper level (or, outer) optimization problem is constrained by another, lower level (or, inner) optimization problem , BLP problems occur in diverse applications, such as transportation, economics, ecology, engineering and others. They have been extensively studied in the literature [1-7], E. Roghanian, et, al.[8] , Integrated goal programming, kuhn-tucker conditions, and penalty function approaches to solve linear BLP problems, K¨oppe et al.[9] developed a parametric integer programming approach for problems with pure integer lower-level problems. M.S. Osman, et al. [10], presented A solution methodology of bi-level linear programming based on genetic algorithm, Sakawa et al [11] proposed a interactive fuzzy programming for various bi-level programming problems.

In a recent work, multi-level decision making models, are used to character decentralized decision making problems where decision makers are in a hierarchical organization[12], G. Zhang et al. [13] presented a general tri-level decision making model, defined the solution concept of the model and developed a Kth-best algorithm to solve the TLDM model, Lu et al. [14] proposed a multi-follower tri-level decision making framework, Tri- level Linear Programming Problems(TLPPs) have been extensively studied in the literature [15-17] . M.S. Osman, et al.[18] presented a compromise weighted solution for multilevel programming problems (MLPPs), where a non-dominated solution set is obtained. Sinha [19, 20] studied alternative MLP techniques based on fuzzy mathematical programming (FMP) , in such techniques the last (lower) level is the most important, and the decision of the lowest level remains either unchanged or closest to individual best decisions, which leads to the decision power of the lowest level DM dominates the higher level DM. To overcome such difficulties, the fuzzy goal programming (FGP) approach to multi-decision-making problems was introduced by Mohamed [21] which is extended by Pramanik and Roy [22] to solve MLPPs. Recently, Lachhwani K., et,al [23] showed a procedure for solving multilevel fractional programming problems in a large hierarchical decentralized organization using fuzzy goal programming approach.

The aim of this paper is to present an interactive fuzzy goal programming approach to Tri-level linear programming problems obtaining the preferred compromise solution and its corresponding aspiration levels, it makes an extension work of Waiel .F. Abd El-Wahed, et.al. [24]. The proposed method has an advantage that candidates for a satisfactory solution can be easily obtained through the combined use of the interactive fuzzy programming, and goal programming. In the proposed methodology the fuzzy goal levels of each objective are involved, which are determined by individual optimal solutions. Then, the fuzzy goals are characterized by the associated membership functions, which are maximized. The approach obtains an efficient solution which is close to the best bound of the higher levels DMs, by the means of introducing negative and positive deviational variables and assigning a higher aspiration level to each objective function via updating both the membership values and the aspiration levels.

To illustrate the effectiveness of the proposed approach, two numerical examples are solved and compare the results with the change in tolerance limits.

The paper is organized as follows: the Formulation of MLPP, and the related terminology are presented in section 2, the interactive fuzzy goal programming for solving MLPP is presented in Section3, in section 4. The steps of the proposed approach are presented, two numerical illustrative examples and a short discussion are presented in section 5. The paper will be finalized with its Conclusion and References.

II. Formulation of MLPP

Hierarchical optimization or (MLPPs), have the following common characteristics[23]: interactive decision making units exist within predominantly hierarchical structures; the execution of decision is sequential from higher level to lower level; each decision making unit independently controls a set of decision variables and is interested in maximizing its own objective but is affected by the reaction of lower level decision makers (DMs). Due to their dissatisfaction with the decision of the higher level DMs, decision deadlock arises frequently in the decision-making situation.

Consider a TLP problem of maximization-type objectives at each level. Mathematically, it can be formulated as follows [25]:

$$\begin{aligned}
&\max_{x_1}\ f_1(X) = c_{11}x_1 + c_{12}x_2 + c_{13}x_3\,,\\
&\text{Where,} x_2 \text{ and } x_3 \text{ solve:}\\
&\max_{x_2}\ f_2(X) = c_{21}x_1 + c_{22}x_2 + c_{23}x_3\,,\\
&\text{Where,} x_3 \text{ solves:}\\
&\max_{x_3}\ f_3(X) = c_{31}x_1 + c_{32}x_2 + c_{33}x_3\\
&\quad s.t. \quad A_1x_1 + A_2x_2 + A_3x_3 \le b,\\
&\qquad\qquad x_1, x_2, x_3 \ge 0.
\end{aligned} \tag{1}$$

Where, $X = (x_1, x_2, x_3)$ denote the decision variables under control of DM1, DM2 and DM3 respectively. For $i = 1,2,3$, x_i is n_i- dimensional decision variable, and $f_i(x)$ is the related objective function to 1^{st}, 2^{nd}, and 3^{rd} level, respectively.

Let $X = x_1 \cup x_2 \cup x_3$ and $n = n_1 + n_2 + n_3$ then, c_{11}, c_{21}, c_{31} are constant row vectors of size $(1\times n_1)$, c_{12}, c_{22}, c_{32} are of size $(1\times n_2)$ and c_{13}, c_{23}, c_{33} are of size $(1\times n_3)$, b is an m -dimensional constant column vector, and A_i is an $m \times n_i$ constant matrix. Each DM has to improve his strategy from a jointly dependent set S :

$$S = \{X |\ A_1x_1 + A_2x_2 + A_3x_3 \le b,\ x_1, x_2, x_3 \ge 0\}.$$

III. The Interactive Fuzzy Goal Programming formulation of TLPP

In order to consider the imprecise nature of the DMs' judgments for the objectives, assume that the DMs have fuzzy goals for each of the objective functions in the Tri-level linear programming problem, such as " $f_i(X),\ i = 1,2,3$ should be substantially greater than or equal to some specific value", thus, the objective functions are to be characterized by the associated membership functions. In this paper, the top level DMs specify fuzzy goals and an aspiration levels to each of them

Fuzzy sets theory has been implemented in mathematical programming since 1970 when Bellman and Zadeh [26] introduced the basic concepts of fuzzy goals G, fuzzy constraints C, and fuzzy decisions D. Based on these concepts, the fuzzy decision is defined as $D = G \cap C$ which is characterized by the following membership function : $\mu_D(x) = \min(\mu_G(x), \mu_C(x))$.

let us describe the fuzzy goals of TLP problem, assuming that $DM_i \ ,i=1,2,3$ selects the following linear membership function $\mu_i(F_i(X))$, which is a strictly monotonic increasing function:

$$\mu_i(F_i(X))=\begin{cases}1\ , & if \quad F_i \geq F_i^{\max} \\ \dfrac{F_i - F_i^{\min}}{F_i^{\max} - F_i^{\min}}\ , & if \quad F_i^{\min} < F_i < F_i^{\max} \\ 0\ , & if \quad F_i \leq F_i^{\min}\end{cases} \qquad (2)$$

where $F_i^{\max}$ is the best upper bound and $F_i^{\min}$ is the worst lower bound of the objective function i, respectively. They are calculated as follows:

$F_i^{\max} = \max\limits_X F_i(X), \quad s.t. \ \ S$ and $F_i^{\min} = \min\limits_X F_i(X), \quad s.t. \ \ S$

It is assumed that the first level DM and the second level DM determine the aspiration levels $\hat{Z}_l,\ l=1,2$. By using the given linear membership functions and following the fuzzy decision of Bellman and Zadeh [26],Then, the TLP problem (1) can be represented as follows:

$$\begin{aligned}&\max\{\min \mu_i(F_i(X))\},\ i=1,2,3 \\ &s.t. \quad X \in S\end{aligned} \qquad (3)$$

By introducing an auxiliary variable λ, problem (3) can be transformed into the following linear programming model:

$$\begin{aligned}&Max\ \lambda \\ &s.t. \\ &\mu_1(F_1(X)) \geq \lambda, \\ &\mu_2(F_2(X)) \geq \lambda, \\ &\mu_3(F_3(X)) \geq \lambda, \\ &X \in S\end{aligned} \qquad (4)$$

The interactive process terminates if the following two conditions are satisfied:

1. If $\mu_1(F_1(X)) \geq \hat{Z}_1\ , and\ \mu_2(F_2(X)) \geq \hat{Z}_2$.
2. The ratios $\Delta_1 = \dfrac{\mu_2(F_2(X))}{\mu_1(F_1(X))}$, and $\Delta_2 = \dfrac{\mu_3(F_3(X))}{\mu_2(F_2(X))}$ of satisfactory degrees in the closed interval between its lower and its upper bounds specified by the first DM and the second DM respectively.

Otherwise, for the dissatisfying upper levels DM , the problem (4) is re formulated as an interactive fuzzy goal programming model [24], let us introduce the following positive and negative deviational variables:

$$F_l(X) - d_l^+ + d_l^- = Z_l\ ,d_l^+, d_l^- \geq 0, \quad l=1,2 \qquad (5)$$

Lachhwani K., et,al [23] considered over deviation from any fuzzy goal implies the full achievement of the desired values, so the proposed approach assigns only negative deviational variables to the achievement function and minimize negative deviational variables to get a compromise optimal solution. Then equation (5) can be written as follow:

$$F_l(X) + d_l^- \geq Z_l\ , \qquad l=1,2 \qquad (6)$$

Then the membership functions are improved by means of changing the tolerances of the objectives. Such alternative membership functions during a solution process reflect the progressive preference.

With the improved membership functions $\mu_l'(F_l(X))$, and the constraints described in (6), the following problem will be formulated:

$$
\begin{aligned}
&Max\ \lambda \\
&s.t. \\
&F_l(X)+d_l^- \geq Z_l\ , \\
&\mu_l'(F_l(X)) \geq \lambda,\ l=1,2 \\
&\mu_3(F_3(X)) \geq \lambda, \\
&X \in S, d_l^- \geq 0\ ,\ d_l^- \leq \lambda
\end{aligned}
\qquad (7)
$$

If an optimal solution to problem (7) exists, it follows that the first and the second DMs obtain a satisfactory solution. Then solution procedure of the TLP problem (1) can be summarized in the following steps:

IV. The solution procedure

Step 1: Develop the TLP as described in problem (1).
Step 2: Calculate the individual minimum and maximum of each objective function in the three levels under the given constraints.
Step 3: Ask each DM to determine the best lower bound and the worst upper bound.
Step 4: Define the membership function of each objective function, the initial aspiration levels $\hat{Z}_l$, and also the closed intervals for $\Delta_l\ , l=1,2$.
Step 5: Set $k=1$. Solve the maximum problem (4) using MATLAB program for obtaining an optimal solution which maximizes the smaller degree of satisfaction between those of the three DMs.
Step 6: Calculate $\mu_i\left(F_i\left(X^k\right)\right)$, and $\Delta_i^k = \dfrac{\mu_{i+1}\left(F_{i+1}\left(X^k\right)\right)}{\mu_i\left(F_i\ \left(X^k\right)\right)}$, $i=1,2,3$
Step 7: The interactive process terminates if $\mu_l\left(F_l\left(X\right)\right) \geq \hat{Z}_l\ ,\ \Delta_l \in \left[\Delta_{l1},\ \Delta_{l2}\right],\ l=1,2$.
Then the upper DMs are satisfied with the optimal solution to problem (4), the optimal solution becomes a satisfactory solution . Otherwise, go to step 8.
Step 8: ask the dissatisfying DMs to determine a new aspiration levels.
Step 9: Construct an improved membership functions $\mu_l'\left(F_l\left(X\right)\right)$, with these new tolerances: $F_1^{best} = \hat{Z}_1$, and $F_1^{worst} = F_1^k$.
Step 10: Set $k=k+1$. With $\mu_l'\left(F_l\left(X\right)\right)$, and constraints (6), solve problem (7) using MATLAB code to get the preferred compromise solution to the TLP. If the current solution $\left(X^k\right)$ satisfies the termination conditions and the upper DMs accept it, then the approach stops and the current solution becomes a satisfactory solution. Otherwise, go to step 8.

V. Illustrative Examples

5.1 Example 1: Consider the following numerical TLP problem [27],

$$\max_{x_1} f_1(X) = 7x_1 + 3x_2 - 4x_3,$$
$$\text{Where}, x_2 \text{ and } x_3 \text{ solve}:$$
$$\max_{x_2} f_2(X) = x_2,$$
$$\text{Where}, x_3 \text{ solves}:$$
$$\max_{x_3} f_3(X) = x_3$$
$$\begin{aligned} s.t.:\quad & x_1 + x_2 + x_3 \le 3, \\ & x_1 + x_2 - x_3 \le 1, \\ & x_1 + x_2 + x_3 \ge 1, \\ & -x_1 + x_2 + x_3 \le 1, \\ & x_3 \le 0.5, \\ & x_1, x_2, x_3 \ge 0. \end{aligned} \tag{8}$$

Set $k = 1$. The individual best and worst solutions subject to the system constraints are shown in table 1. Suppose that the first DM and the second DM specify the initial aspiration levels, the lower and the upper bound of the ratio of satisfactory degrees as : $\hat{Z}_1 = 8.5$, $\hat{Z}_2 = 1.0$, , $\Delta_1 = [0.8,1]$, and $\Delta_2 = [0.7,1]$ respectively.

Based on table(1), the DMs identify the linear membership function (2), developing the maximum problem (4) as follow:

$$\begin{aligned} & Max\ \lambda \\ & s.t. \\ & 7x_1 + 3x_2 - 4x_3 + 0.5 \ge 9\lambda, \\ & x_2 \ge \lambda, \\ & x_3 \ge 0.5\lambda, \\ & x_1 + x_2 + x_3 \le 3, \\ & x_1 + x_2 - x_3 \le 1, \\ & x_1 + x_2 + x_3 \ge 1, \\ & -x_1 + x_2 + x_3 \le 1, \\ & x_3 \le 0.5, \\ & x_1, x_2, x_3 \ge 0. \end{aligned} \tag{9}$$

The obtained result of the first iteration is shown at the column labeled "1st" in Table 2. For the obtained optimal solution $(x_1^1, x_2^1, x_3^1) = (0.8077, 0.6923, 0.5)$ to problem (9), corresponding membership function values, and the ratio of satisfactory degrees are calculated.

As $F_1^1 < 8.5$,so the first DM is not satisfied with this solution, assume that he/she defines a new aspiration level as $\hat{Z}_1' = 8.5$. Let $F_1^{best} = 8.5$, and $F_1^{worst} = 5.7308$, construct an improved membership function.

-Set $k = 2$. Let, $\mu_1'(F_1(X)) = \dfrac{(7x_1 + 3x_2 - 4x_3 - 5.7308)}{8.5 - 5.7308}$, and $7x_1 + 3x_2 - 4x_3 + d_1^- \ge 8.5$.

- Then problem (7) can be developed as follow:

$$
\begin{aligned}
& Max\ \lambda \\
& s.t. \\
& 7x_1 + 3x_2 - 4x_3 + d_l^- \geq 8.5, \\
& 7x_1 + 3x_2 - 4x_3 - 5.7308 \geq 2.77\lambda, \\
& x_2 \geq \lambda, \\
& x_3 \geq 0.5\lambda, \\
& x_1 + x_2 + x_3 \leq 3, \\
& x_1 + x_2 - x_3 \leq 1, \\
& x_1 + x_2 + x_3 \geq 1, \\
& -x_1 + x_2 + x_3 \leq 1, \\
& x_3 \leq 0.5, \\
& x_1, x_2, x_3 \geq 0, \\
& d_1^- \geq 0\ ,\ d_1^- \leq \lambda
\end{aligned}
\tag{10}
$$

For the obtained optimal solution $\left(x_1^2, x_2^2, x_3^2\right) = (1.5, 0.0, 0.5)$ to problem (10), as shown at the column labeled "2 nd" in Table 2, is the preferred solution to the first and the third DMs . But the second DM is not satisfied with this solution, so the approach will aid the unsatisfied DM to improve his/her solution. Assume that the second DM defines a new aspiration level as $\hat{Z}_1' = 1$. Let $F_2^{best} = 1$, and $F_2^{worst} = 0.6923$, construct an improved membership function.

-Set $k = 3$. Let, $\mu_2'\left(F_2(X)\right) = \dfrac{(x_2 - 0.6923)}{1 - 0.6923}$, and $x_2 + d_2^- \geq 1.0$.

Adding the new constraints , and considering the first DM insist on his preferences.

Then problem (10) can be rewritten as follow:

$$
\begin{aligned}
& Max\ \lambda \\
& s.t. \\
& 7x_1 + 3x_2 - 4x_3 + d_l^- \geq 8.5, \\
& 7x_1 + 3x_2 - 4x_3 - 5.7308 \geq 2.77\lambda, \\
& x_2 + d_2^- \geq 1.0, \\
& x_2 - 0.6923 \geq 0.31\lambda, \\
& x_3 \geq 0.5\lambda, \\
& x_1 + x_2 + x_3 \leq 3, \\
& x_1 + x_2 - x_3 \leq 1, \\
& x_1 + x_2 + x_3 \geq 1, \\
& -x_1 + x_2 + x_3 \leq 1, \\
& x_3 \leq 0.5, \\
& x_1, x_2, x_3 \geq 0, \\
& d_1^- \geq 0\ ,\ d_1^- \leq \lambda, \\
& d_2^- \geq 0\ ,\ d_2^- \leq \lambda
\end{aligned}
\tag{11}
$$

-The interaction process results are shown in table 2.

Since the 1st and the 2nd DMs are satisfied with the 3rd iteration, as the values of the two objective functions increase and converge toward the ideal solution. Then the satisfactory solution is obtained and the interaction procedure is terminated.

	1^{st} Level DM	2^{nd} Level DM	3^{rd} Level DM
F^{best}	8.5 at $(1.5, 0, 0.5)$	1 at $(0, 1.0, 0)$	0.5 at $(0.25, 0.25, 0.5)$
F^{worst}	-0.5 at $(0, 0.5, 0.5)$	0 at $(0.5, 0, 0.5)$	0 at $(0.5, 0.5, 0)$

Table 1. The individual best and worst solutions at all levels of Example(1).

Interaction	1 st	2 nd	3 rd
λ	0.69	0	0.3
x_1	0.8077	1.5	1.0217
x_2	0.6923	0	0.6771
x_3	0.5	0.5	0.5
F_1	5.7308	8.5	7.1832
F_2	0.6923	0.0	0.7
F_3	0.5	0.5	0.5
$\mu_1(F_1)$	0.69	1.0	0.85
$\mu_2(F_2)$	0.69	0.0	0.7
$\mu_3(F_3)$	1.0	1.0	1.0

Table 2. Example 1. results of the proposed approach.

Performance analysis

To evaluate the performance of the proposed approach, let us consider the solution of the illustrative example by using different approaches [13,18,27,28].

To determine the degree of closeness of the proposed approach results to the ideal solution, let us define the following distance functions [29]:

$$D_\infty(\lambda, K) = \max_K \{\lambda_K (1 - d_K)\}$$

where, in maximization problems, d_K takes the form: $d_K = F^{Compromize} / F^{Optimal}$,

Where : $F^{Optimal}$ is the optimal solution of F_k , and

$F^{Compromize}$ is the preferred compromise solution

Thus, we can state that the approach which can derive a preferred compromise solution is better than the others if: $\min D_\infty(\lambda, K)$ is achieved by its solution.

Based on this measure, Table 3 summarizes the results of the five approaches . In the given example, it is assumed that $\lambda_1 = 0.4,\ \lambda_2 = 0.3, \lambda_3 = 0.3$.

From Table 3, it is clear that the proposed approach in this paper gave a preferred compromise solution which is the best solution for D_∞ . Note that the achieved compromised interactive solutions may be the same obtained by other approaches or around them.

	The Proposed Approach	**Weighting approach[18]**		**Zhang et al. [13]**	**Shih et al. [27]**	**Sakawa et al. [28]**
		Case(1)	**Case(2)**			
F_1	7.1832	4.5	6.5	4.5	6.18	5.8650
F_2	0.7	1.0	0.5	1.0	0.58	0.65
F_3	0.5	0.5	0.5	0.5	0.5	0.5
$\mu_1(F_1)$	0.85	0.5294	0.7647	0.5294	1.0	0.69
$\mu_2(F_2)$	0.7	1.0	0.5	1.0	1.0	0.65
$\mu_3(F_3)$	1.0	1.0	1.0	1.0	1.0	1.0
D_∞	0.09	0.1882	0.15	0.1882	0.126	0.124

Table 3. Comparison of example.1 solutions by four different approaches

5.2 Example 2: Consider the following numerical TLP problem [18,20,27,30]:

$$\max_{x_1,x_2} f_1(X) = 7x_1 + 3x_2 - 4x_3 + 2x_4,$$

Where, x_3 and x_4 solve:

$$\max_{x_3} f_2(X) = x_2 + 3x_3 + 4x_4,$$

Where, x_4 solves:

$$\max_{x_4} f_3(X) = 2x_1 + x_2 + x_3 + x_4$$

$$\begin{aligned} s.t.: \quad x_1 + x_2 + x_3 + x_4 &\le 5, \\ x_1 + x_2 - x_3 - x_4 &\le 2, \\ x_1 + x_2 + x_3 &\ge 1, \\ -x_1 + x_2 + x_3 &\le 1, \\ x_1 - x_2 + x_3 + 2x_4 &\le 4 \\ x_1 + 2x_3 + 3x_4 &\le 3, \\ x_4 &\le 2, \\ x_1, x_2, x_3, x_4 &\ge 0. \end{aligned} \tag{12}$$

Set $k = 1$. The individual best and worst solutions subject to the system constraints are shown in table 4. Suppose that the three level DMs specify their initial aspiration levels, the lower and the upper bound of the ratio of satisfactory degrees as : $\hat{Z}_1 = 16, \hat{Z}_2 = 5, \hat{Z}_3 = 4$, , $\Delta_1 = [0.8,1]$, and $\Delta_2 = [0.7,1]$ respectively.

Based on table (4), the DMs identify the linear membership function, and problem(4) is solved with these data using MATLAB.

The obtained result of the first iteration is shown at the column labeled “1st” in Table 5. Assume that the upper level DMs are not satisfied, and want to improve their objective functions.

So the improved upper level membership functions, and the aspiration levels constraints are constructed as follow:

-Set $k=2$. Let, $\mu_1'(F_1(X))=\dfrac{(7x_1+3x_2-4x_3+2x_4-13.1754)}{16.25-13.1754}$,

$\mu_2'(F_2(X))=\dfrac{(x_2+3x_3+4x_4-4.2408)}{5-4.2408}$, $7x_1+3x_2-4x_3+2x_4+d_1^-\geq 16.25$, and

$x_2+3x_3+4x_4+d_2^-\geq 5$. Then problem (7) can be developed and solved.

-The interaction process results are shown in table 5.

Since the 1st and the 2nd DMs are satisfied with the 2nd iteration, as the values of the two objective functions increase and converge toward the ideal solution. Then the satisfactory solution is obtained and the interaction procedure is terminated.

	1^{st} Level DM	2^{nd} Level DM	3^{rd} Level DM
F^{best}	16.25 at $(2.25,0,0,0.25)$	5 at $(0,1,0,1)$	5 at $(1.7131,0.9303,0.6434,0)$
F^{worst}	-4 at $(0,0,1,0)$	0 at $(1,0,0,0)$	1 at $(0,0.5,0.5,0)$

Table 4. The individual best and worst solutions at all levels of Example(2).

Interaction	1st	2nd
λ	0.8482	0.7635
x_1	1.0506	1.212
x_2	1.6204	1.64
x_3	0.0637	0
x_4	0.6073	0.694
F_1	13.1754	14.7884
F_2	4.2408	4.4156
F_3	4.3927	4.7569
$\mu_1(F_1)$	0.8482	0.9278
$\mu_2(F_2)$	0.8482	0.8831
$\mu_3(F_3)$	0.8482	0.9392

Table 5. Example 2. results using the proposed approach.

Based on the D_∞ measure, Table 6 summarizes the results of the five approaches . In the given example, it is assumed that $\lambda_1=0.4,\ \lambda_2=0.3, \lambda_3=0.3$.

From Table 6, it is clear that the proposed approach in this paper gave a preferred compromise solution which is the best solution for D_∞.

.

	The Proposed Approach	**Weighting approach[18]**	**Sinha [20]**	**Shih et al. [27]**	**Surapati et al. [30]**
F_1	14.7884	16.25	12.01	11.70	13.00
F_2	4.4156	1.0	3.18	3.02	4.71
F_3	4.7569	4.75	4.94	4.94	4.28
D_∞	0.036	0.24	0.11	0.112	0.08

Table 6. Comparison of example.2 solutions by four different approaches

VI. Conclusion:

In this paper, we have proposed an interactive fuzzy goal programming approach for tri-level linear programming problems. One of the most important features of our approach is to provide satisfactory (near optimal) solution efficiently by updating the aspiration levels, and the membership functions of decision makers at the upper level with considerations of overall satisfactory balance among all levels, where it can produces solutions which are improved to the results obtained by most of the other existing approaches. Finally, illustrative numerical examples for three-level linear programming problems have been have been successfully solved to demonstrate the feasibility of the proposed approach. The performance of the proposed approach was evaluated by using of metric distance functions with other applied approaches.

References:

[1] Nataliya I. Kalashnykova, Vyacheslav V. Kalashnikov, Stephan Dempe1 and Aaron Arevalo Franco, "Application of a Heuristic Algorithm to Mixed-Integer Bi-level Programming Problems", International Journal of Innovative Computing, Information and Control , ISSN 1349-4198, V. 7, N. 4, pp. 1819-1829, 2011.

[2] B. Colson, P. Marcotte and G. Savard, "An Overview of Bilevel Optimization," Annals of Operational Research, Vol. 153, No. 1, pp. 235-256, 2007.

[3] C. O. Pieume, L. P. Fotso and P. Siarry, "A Method for Solving Bilevel Linear Programming Problem," Journal of Information and Optimization Science, Vol. 29, No. 2, pp. 335-358,2008.

[4] G. Zhang, Y. Gao, and J. Lu. Competitive strategic bidding optimization in electricity markets using bilevel programming and swarm technique. IEEE Transactions on Industrial Electronics, 58(6):2138–2146, 2011.

[5] Y. Gao, G. Zhang, J. Lu, and H. M. Wee. Particle swarm optimization for bi-level pricing problems in supply chains. Journal of Global Optimization, 51(2):245–254, 2011.

[6] Jianzhong Wang and Gang Du, A solution to interval linear bi-level programming and its application in decentralized supply chain planning, IEEE, 2035-2038, 2008.

[7] P. Lasunon, J. Wetweerapong and T. Remsungnen, A new algorithm for solving bi-level linear programming problems, Proceeding of the 16th Annual Meeting in Mathematics, Thailand, 59-66, 2011.

[8] E. Roghanian, M.B. Aryanezhad and S.J. Sadjadi, " Integrating goal programming, kuhn-tucker conditions, and penalty function approaches to solve linear bi-level programming problems", Applied Mathematics and Computation, 585-590, 2008.

[9] K¨oppe, M., M. Queyranne, and C. Ryan. A parametric integer programming algorithm for bi-level mixed integer programs, 2009. Available at http://arxiv.org/abs/0907.1298v2.

[10] M.S. Osman, W.F. Abd El-Wahed, Mervat M.K. El Shafei and Hanaa B.Abd El Wahab, "A solution methodology of bi-level linear programming based on genetic algorithm", Journal of Mathematics and Statistics, 5(4), 352-359, 2009.

[11] M.Sakawa and I.Nishizaki. Interactive fuzzy programming for decentralized two-level linear programming problems, Fuzzy Sets and Systems, 125:301-315, 2002.

[12] Zhen Zhang, Guangquan Zhang, Jie Lu, and Chonghui Guo, " A fuzzy tri-level decision making algorithm and its application in supply chain" 8th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), 2013.

[13] G. Zhang, Jie Lu, Javier Montero and Yi Zeng, Model, solution concept, and Kth best algorithm for linear trilevel programming, Information Science, 481-492. Int. J. Pure Appl. Sci. Technol., 7(2) , 149-157, 2010.

[14] J. Lu, G. Zhang, J. Montero, and L. Garmendia. Multi-follower tri-level decision making models and system. IEEE Transactions on Industrial Informatics, 8(4):974–985, 2012.

[15] P. Lasunon, and T. Remsungnen, A new algorithm for solving Tri-level linear programming problems, International Journal of Pure and Applied Sciences and Technology, ISSN 2229 – 6107, 7(2), pp. 149-157, 2011 . Available online at www.ijopaasat.in

[16] P. Faisca Nuno, M. Saraiva Pedro, Berd Rustem and N. Pistikopoulos Efstratios, A multiparametric programming approach for multilevel hierarchical and decentralised optimization problems, Springer, (2007).

[17] G.Z.Ruan, S.Y.Wang, Y.Yamamoto and S.S.Zhu, Optimality conditions and geometric properties of a linear multilevel programming problem with dominated objective functions, Journal of Optimization Theory and Applications, 123(2), 409-429, 2004.

[18] Osman M. S., Gadalla M. H., Zeanedean R. A., Rabie R. M. , A Compromise Weighted Solution For Multilevel Linear Programming Problems, Int. Journal of Engineering Research and Applications, Vol. 3, Issue 6, pp.927-936, 2013.

[19] S. Sinha, Fuzzy Mathematical Programming Applied to Multi-Level Programming Problems, Computers and Operations Research, 30 1259–1268, (2003)a.

[20] S. Sinha, Fuzzy Programming Approach to Multi-Level Programming Problems, Fuzzy Sets and Systems, 136 , 189–202, (2003)b.

[21] Mohamed RH , The relationship between goal programming and fuzzy programming. Fuzzy Sets and Systems 89:215–222, 1997.

[22] Pramanik S, Roy TK , Fuzzy goal programming approach to multilevel programming problems. European Journal of Operational Research 176:1151–1166, 2007.

[23] Lachhwani and Poonia, Mathematical solution of multilevel fractional programming problem with fuzzy goal programming approach. Journal of Industrial Engineering International, doi:10.1186/2251-712X-8-16 , 8-16, 2012.

[24] Waiel F. Abd El-Wahed, Sang M. Lee, Interactive fuzzy goal programming for multi-objective transportation problems, www.elsevier.com/locate/omega , omega 34 , 158 – 166, 2006.

[25] G. Anandalingam and T.L. Friesz, Hierarchical Optimization: an Introduction, Annals of Operations Research, 34 (1), 1–11, 1992.

[26] Belmann R, Zadeh L. Decision making in a fuzzy environment. Management Science , 17B:141–64, 1970.

[27] H.S. Shih, Y.J. Lai and E.S. Lee, Fuzzy Approach for Multi-Level Programming Problems, Computers and Operations Research, 73–91, 23 (1) (1996).

[28] M. Sakawa, I. Nishizaki and Y. Uemura, Interactive Fuzzy Programming for Multi-Level Linear Programming Problems, Computers & Mathematics with Applications, 71–86, 36 (2) (1998).

[29] Steuer R. Multiple criteria optimization: theory, computation, and application. New York: Wiley; 1986.

[30] P. Surapati and K. R. Tapan, Fuzzy Goal Programming Approach to Multi-level Programming Problems, European Journal of Operational Research, 1151–1166, 176 (2007).